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MOVEMENT STABILITY ANALYSIS OF A PIPE STRING IN A THIXOTROPIC FLUID

I. Sh. Akhatov, M. M. Khasanov, and I. G. Khusainov

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Features of movement of a drill pipe string in a well filled with a thixotropic fluid are analyzed taking into account inertial and elastic forces of the rope system. It is shown that processes of fluid structure degradation can lead to nonstationary movement of the pipes. In this case periodic and chaotic auto-oscillations are excited in the system.

Introduction. In the practice of drilling it has been noticed that various complications associated with a change in the hydrodynamic pressure of the drilling mud on the well walls can occur when a drill pipe string is lowered or lifted [1–8]. When a drilling (or casing) string is lowered into a well, a portion of the drilling mud is displaced into the annular space between the string and the well walls, which leads to the appearance of a pressure gradient required to overcome inertial and frictional forces. Thereby an excess hydrodynamic pressure is added to the hydrostatic one, which can lead to hydraulic breakdown of the stratum and entry the drilling mud into the cracks formed. In certain cases the pipes can be crumpled and the valves damaged. On the othehand, when the pipes are lifted from the well, pressure in the well is reduced, which can result in inflow of stratum fluids into the well, and, consequently, in emergency gushing or collapse of the well.

The calculation of the hydrodynamic pressures occurring during lifting/lowering operations is reduced to an analysis of fluid movement between two coaxial cylinders when one of them (the inner one) moves with a certain velocity. In the case of steady movement of viscous and viscoplastic fluids this problem has been solved in [9–11]. Numerous theoretical and experimental studies show that the magnitude of the hydrodynamic pressure is affected substantially by inertial forces appearing during transient movement of a string [1–3, 6–8]. This effect is very dangerous since accelerated movement of the string can result in pressure pulsations that produce high peak loads on the well walls.

Drilling fluids are thixotropic media. In formulating the problem one should take into account degradation/restoration of the drilling mud structure. Linear kinetic equations were employed in [12, 13] to simulate degradation/restoration processes. It has been shown that various characteristics of steady-state movement of rheological fluids can be described satisfactorily within the scope of this approach. For describing the degradation/restoration processes, in [14] we proposed using nonlinear models that are a generalization of classical models of the "predator-prey" type. It was noted that in addition to describing the steady-state characteristics complex transient (auto-oscillation and chaotic) modes can be described by using nonlinear models. In [15] a simple nonlinear kinetic equation was proposed for describing the processes of degradation/restoration of bonds between structural elements of a medium. This equation is employed for simulating fluid movement in the clearance between the cylinders of a rotation viscometer.

The present work is devoted to analysis of string movement stability during lowering/lifting operations. It is shown that processes of degradation/restoration of drilling mud structure can result in loss of stability of stationary string movement and establishment of periodic or chaotic auto-oscillations that manifest themselves in dangerous pulsations of hydrodynamic pressure. Results that make it possible to identify justified regimes of descending/lifting operations that prevent excitation of these oscillations are obtained.

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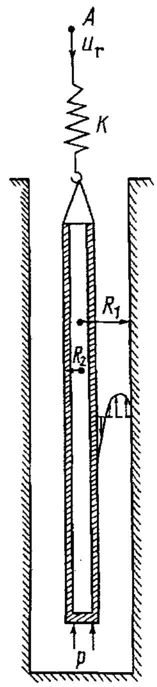

Fig. 1. Schematic diagram of drilling string movement.

1. Formulation of the Problem. Let a pipe string that is closed at the bottom move down a well of radius R_1 filled with a thixotropic fluid. The outer radius of the pipes is R_2. Then the velocity distribution pattern of the fluid in the annular space between the well and the pipes has the shape shown in Fig. 1 (in the case of lifting of the string the fluid movement pattern will be the opposite). Let u_r be the speed of the rope leaving the drum of the drilling winch. This speed is determined by the driller and, thus, may be considered to be a specified function of time: $u_r = f(t)$. The speed of the string can be substantially different from u_r since the ropes from which the string hangs are elastic (it is well known that the amplitude of the string speed oscillations caused by the elasticity of the rope system can be as high as 30% of the rope speed [8]). Taking this fact into account, let us introduce a spring, located between the rope end leaving the winch drum (point A) and the upper end of the drilling string, into the diagram presented in Fig. 1. To complete the model we can assume that this spring simulates the elasticity of both the rope system and the pipe string.

The equation of string movement has the form

$$M \frac{du}{dt} = F + \Delta p Q + Mg - k(x_1 - \xi). \tag{1}$$

The displacement on the rope ξ is determined by the equation

$$\frac{d\xi}{dt} = f(t).$$

Often the rope speed (the function $f(t)$) is assumed to be constant in estimating the hydrodynamic pressure. A more exact approach requires taking into account that the period of lowering a linkage of several pipes is divided into three steps: acceleration, movement with constant speed, and deceleration.

If $f(t) = u_0 = \text{const}$, then

$$\frac{du}{dt} = \frac{d^2}{dt^2}(x_1 - \xi) = \frac{d^2 x}{dt^2},$$

where $x = x_1 - \xi$ is the elongation of the spring.

The flow in the annular space can be replaced with a flat flow [6] by writing the fluid movement equation in the form

$$\rho \frac{\partial v}{\partial t} = \frac{\partial}{\partial y}\left(\mu \frac{\partial v}{\partial y}\right) + \frac{\Delta p}{L} + \rho g, \quad 0 < y < h, \tag{2}$$

with the boundary conditions

$$v(0, t) = u, \quad v(h, t) = 0. \tag{3}$$

Drilling fluids are non-Newtonian media whose viscosity decreases with increase in shear rate. This situation takes place due to degradation of structural bonds in shear movement. Using the concentration of degraded bonds s as a quantitative characteristic of the structurization of the fluid, the dependence of the fluid viscosity on the concentration s can be written as follows:

$$\mu(s) = \mu_0 \frac{\exp(-(s/s_*)^n) - \exp(-(s_\infty/s_*)^n)}{1 - \exp(-(s_\infty/s_*)^n)} + \mu_\infty \frac{1 - \exp(-(s/s_*)^n)}{1 - \exp(-(s_\infty/s_*)^n)}. \tag{4}$$

At a concentration of degraded bonds equal to zero the fluid viscosity has its maximum and is equal to $\mu(0) = \mu_0$. With degradation of the bonds (i.e., with increase in s) the viscosity decreases in accordance with a nonlinear exponential law. The minimum of the viscosity $\mu(s_\infty) = \mu_\infty$ is attained when all the bonds are degraded ($s_\infty = 1$).

In order to describe the process of bond degradation/restoration let us introduce the following kinetic equation:

$$\frac{ds}{dt} = -\alpha\left\{s - s_\infty\,[1 - \exp(-\gamma s \mu(s)\,\dot{\varepsilon}^2)]\right\}. \tag{5}$$

In accordance with this equation the concentration of degraded bonds should tend to a certain equilibrium value s_0 that can be obtained from the formula

$$s_0 = s_\infty\,[1 - \exp(-\gamma s_0 \mu(s_0)\,\dot{\varepsilon}^2)].$$

at constant shear rate ($\dot{\varepsilon}$ = const). It is evident from the above formula that with increasing in $\dot{\varepsilon}$ the concentration of degraded bonds increases, tending exponentially to its maximum value s_∞. In addition, at low $\dot{\varepsilon}$

$$s_\infty\,[1 - \exp(-\gamma s \mu(s)\,\dot{\varepsilon}^2)] \approx s_\infty \gamma s \mu(s)\,\dot{\varepsilon}^2,$$

i.e., at low shear rates the rate of bond degradation is directly proportional to the intensity of viscous energy dissipation in the flow.

It is assumed that the fluid is incompressible, and therefore the mass conservation law for the fluid can be written as follows:

$$Qu + 2\pi R_0 \int_0^h v\,dy = 0, \quad R_0 = (R_1 + R_2)/2. \tag{6}$$

We will seek for an approximate solution of Eq. (2) as a parabolic function

$$v = ay^2 + by + c.$$

The coefficients a, b, and c are determined upon substitution of the speed v into Eq. (6) and boundary conditions (3):

$$\pi R_2^2 u + 2\pi R_0 \int_0^h (ay^2 + by + c)\, dy = 0\,, \quad ah^2 + bh + c = 0\,, \quad c = u\,.$$

Solving this system of equations, we obtain

$$v = \left[3\left(\frac{R_2^2}{R_0 h} + 1 \right) \left(\frac{y}{h} \right)^2 - \left(3\frac{R_2^2}{R_0 h} + 4 \right) \frac{y}{h} + 1 \right] u\,,$$

where the speed v satisfies Eq. (2) in the mean, i.e.,

$$\int_0^h \left[\rho \frac{\partial v}{\partial t} - \frac{\partial}{\partial y}\left(\mu \frac{\partial v}{\partial y} \right) - \frac{\Delta p}{L} - \rho g \right] dy = 0\,,$$

whence it follows that

$$\Delta p = -\frac{L}{2}\rho \frac{R_2^2}{R_0 h}\frac{du}{dt} - \frac{L}{h} u \left[\left(3\frac{R_2^2}{R_0 h} + 2 \right) \mu\,(s_2) + \left(3\frac{R_2^2}{R_0 h} + 4 \right) \mu\,(s_1) \right] - L\rho g\,. \tag{7}$$

The functional force of the pipe string in the fluid is given by the expression

$$F = 2\pi R_2 L \left(\mu \frac{\partial v}{\partial y} \right) \Big|_{y=0} = -2\pi R_2 \frac{L}{h}\left(3\frac{R_2^2}{R_0 h} + 4 \right) \mu\,(s_1)\, u\,. \tag{8}$$

Substituting Eqs. (7) and (8) into Eq. (1) and introducing the following dimensionless variables:

$$U = u/u_0\,, \quad \tau = t\alpha\,, \quad S_j = \frac{s_j}{s_*}\,, \quad \eta\,(S_j) = \mu\,(S_j)/\beta\,;$$

$$X = \frac{\alpha}{u_0}\left[\frac{\pi R_2^2 L\rho g - Mg}{k} + x \right],$$

we obtain

$$\frac{d^2 X}{d\tau^2} = -U\,(E_1\,\eta\,(S_1) + E_2\,\eta\,(S_2)) - E_3 X\,, \tag{9}$$

$$\frac{dS_j}{d\tau} = -S_j - A\,(1 - \exp\,(-G_j S_j\,\eta\,(S_j)\,U^2))\,, \tag{10}$$

$$\eta\,(S_j) = Z + \exp\,(-S_j^n)\,, \quad j = 1\,,\, 2\,, \tag{11}$$

where

$$U = 1 + \frac{dX}{d\tau}\,; \quad A = \frac{s_\infty}{s_*}\,; \quad \beta = \frac{\mu_0 - \mu_\infty}{1 - \exp\,(-A^n)}\,;$$

$$E_1 = 2\pi\beta R_2 L\,(2h + R_2)\,(3R_2^2 + 4R_0 h)/B\,;$$

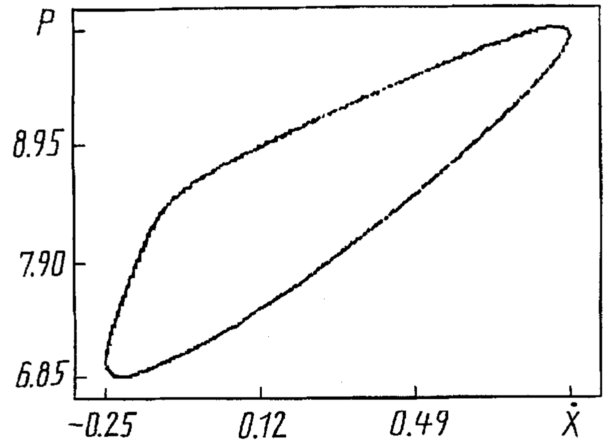

Fig. 2. Phase portrait at $G_1 = 0.4$ when the limiting cycle is achieved.

$$E_2 = 2\pi\beta R_2^2 L\,(3R_2^2 + 2R_0 h)/B\,;$$

$$E_3 = 2R_0 h^3 k/(B\alpha)\,;\quad B = \alpha h^2\,(2MR_0 h + \pi R_2^4 L\rho)\,;$$

$$Z = \frac{\mu_\infty - \mu_0 \exp(-A^n)}{\mu_0 - \mu_\infty}\,;\quad G_1 = \gamma\beta s_*\left(3\frac{R_2^2}{R_0 h} + 4\right)^2\left(\frac{u_0}{h}\right)^2;$$

$$G_2 = \gamma\beta s_*\left(3\frac{R_2^2}{R_0 h} + 2\right)^2\left(\frac{u_0}{h}\right)^2.$$

Thus the simulation problem for pipe string movement in lowering/lifting operations has been reduced to the fourth-order nonlinear dynamic system of equations (9) – (11).

2. Results of Numerical Analysis of the Problem. In order to estimate the coefficients of the system (9) – (11), let us take the following values of the parameters [16]: $R_1 = 0.08$ m, $R_2 = 0.05$ m, $L = 1000$ m, $M = 30{,}000$ kg, $\rho = 1300$ kg/m^3, $k = 5\cdot 10^5$ N/m. Hereinafter $s_* = 0.25$, $\alpha = 0.5$ 1/sec, $n = 10$, $\gamma = 8.5\cdot 10^{-6}$ sec/Pa. Then we obtain $E_1 = 145$, $E_2 = 55$, $E_3 = 45$, $Z = 0.1$, $A = 4$. Let us select the speed of the rope leaving the drum of the drilling winch u_0 ($G_1 \sim u_0^2$) as the controlling parameter.

Calculations shown that the range of the parameter G_1 is divided into three regions differing in the dynamics of the process. In the region $G_1 < 0.3$ the drilling string is lowered with constant speed equal to the speed of the rope leaving the drilling winch drum, and no pulsation of the hydrodynamic pressure is observed. At low G_1 the degraded bonds have time to be restored and the fluid viscosity is equal to its maximum value μ_0.

Within the region $0.3 \le G_1 < 5.8$ (for the speed of the rope leaving the drum this corresponds to the region $0.5 \le u_0 < 2.5$ m/sec) intense degradation of structural bonds takes place and stable oscillations of all variables occur in the system, and the amplitude of the oscillations and the mean value of S_1 are higher than those of S_2 in all cases. The phase portrait in the plane of $\dot{X}$ (the dot denotes the first derivative with respect to time) and the dimensionless pressure P ($P = p/L\rho g$) at $G_1 = 0.4$, when a limiting cycle is achieved, is presented in Fig. 2. As the figure shows, the values of the speed of drilling string lowering and the pressure undergo auto-oscillations. If the value of the pressure is converted from dimensionless to dimensional form, it will range from $87\cdot 10^6$ to $127\cdot 10^6$ N/m^2. At $G_1 = 0.5$ the period is doubled. A further increase in G_1 ($G_1 \ge 0.7$) leads to a complicated limiting cycle. The complication in this case consists in the appearance of an additional "loop" corresponding to a small-scale beat on the background of the regular oscillatory process. At $G_1 = 5.0$ a doubled period with two beats is obtained. The increase in the complication of the oscillations with increase in the controlling parameter results in the appearance of a chaotic mode at $G_1 = 5.3$. The structure of the corresponding strange attractor is gven in Fig. 3. In this case the pressure oscillations occur within the range from $20\cdot 10^6$ to $77\cdot 10^6$ N/m^2.

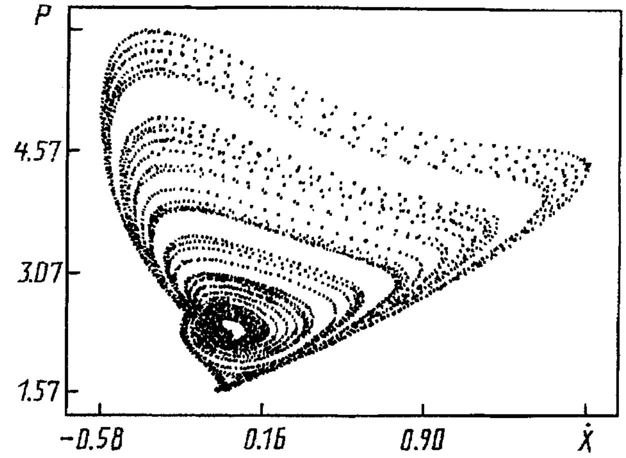

Fig. 3. Strange attractor (G_1 = 5.3).

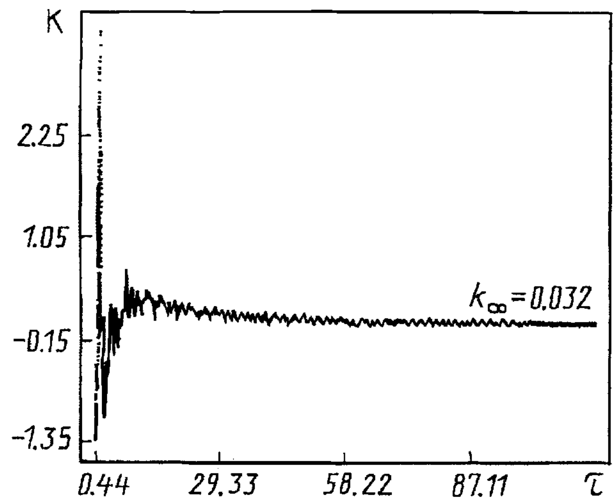

Fig. 4. Dependence of the Kolmogorov entropy on the dimensionless time.

In order to elucidate to what extent the dynamic system under investigation is chaotic at G_1 = 5.3, we calculated the Kolmogorov entropy [17]. The dependence of the Kolmogorov entropy K on the dimensionless time τ is given in Fig. 4. A chaotic process at $\tau \to \infty$ requires a positive value of the entropy. Figure 4 shows that with time the value of the Kolmogorov entropy attains a positive stationary level K_∞ = 0.032.

A further increase in the controlling parameter results in a continuous reduction in the amplitude and number of periods of the oscillations up to vanishing of the oscillatory mode at G_1 = 5.8. The third region of G_1 values ($G_1 \geq 5.8$) is characterized by stable values of all variables as well as by high values of the concentration of degraded bonds S_1 and S_2. An increasing in G_1 to 30 results in complete degradation of the bonds between the structural elements, and the fluid viscosity attains its minimum value. In this region of G_1 values the fluid behaves like an ordinary Newtonian fluid.

Conclusion. An analysis of the stability of pipe string movement in a well filled with a thixotropic fluid has been performed. A model making it possible to account for the inertia of the string, the elasticity of the rope system, and the inertia of the fluid has been suggested. A numerical investigation of the system of differential equations obtained has been performed. The study has shown that movement of the pipes accompanied by intense degradation of the fluid structure, can become unstable with the appearance of periodic and chaotic auto-oscillations that lead to dangerous pulsations of the hydrodynamic pressure on the well walls. The results obtained in the present work are important for understanding the mechanisms of the appearance of complications in the drilling process as well as for justified specification of operating modes in lowering and lifting drill pipes.

NOTATION

M, mass of the drilling string (together with the mass of the drilling mud within the string); F, frictional force of the string in the fluid; Q, area of the string cross section; p, pressure at the lower end of the string; p_{atm}, atmospheric pressure; $\Delta p = p_{atm} - p$; x_1, displacement of the string; g, free-fall acceleration; k, string stiffness simulating the total stiffness of the rope system and the drilling string; u, speed of string movement; t, time; ρ, fluid density; μ, dynamic viscosity of the fluid; v, speed of fluid movement; y, distance from the outer surface of the pipes; L, length of the drilling string; h, clearance between the well and the drilling pipe; s_*, characteristic value of the concentration of degraded bonds at which the fluid viscosity decreases; n, parameter of the fluid characterizing the degree of the dependence of the viscosity on the structurization of the fluid; α and γ, positive constants.

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